Term-II

CHAPTER 13

PROBABILITY

Syllabus

- *Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.*

STAND ALONE MCQs

(1 Mark each)

Q. 1. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then

(A) $A \subset B$ (B) $B \subset A$

(C) $B = \varphi$ (D) $A = \varphi$

Ans. Option (A) is correct.

Explanation:

$$P(A) \neq 0$$

and $$P(B|A) = 1$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$1 = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = P(B \cap A)$$

$$\therefore \quad A \subset B$$

Q. 2. If $P(A|B) > P(A)$, then which of the following is correct :

(A) $P(B|A) < P(B)$

(B) $P(A \cap B) < P(A).P(B)$

(C) $P(B|A) > P(B)$

(D) $P(B|A) = P(B)$

Ans. Option (C) is correct.

Explanation:

$$P(A|B) > P(A)$$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} > P(A)$$

$$\Rightarrow \quad P(A \cap B) > P(A).P(B)$$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(A)} > P(B)$$

$$\Rightarrow \quad P(B|A) > P(B)$$

Q. 3. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(A) $P(B|A) = 1$ (B) $P(A|B) = 1$

(C) $P(B|A) = 0$ (D) $P(B|A) = 0$

Ans. Option (B) is correct.

Explanation :

$$P(A) + P(B) - P(A \text{ and } B) = P(A)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow \quad P(B) - P(A \cap B) = 0$$

$$\Rightarrow \quad P(A \cap B) = P(B)$$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Q. 4. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1} (B) $\left(\frac{1}{2}\right)^5$

(C) $\left(\frac{9}{10}\right)^5$ (D) $\frac{9}{10}$

Ans. Option (C) is correct.

Explanation : The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denotes the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb,

$$p = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{10}$.

$$\therefore \quad P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x \left(\frac{9}{10}\right)^{5-x} \left(\frac{1}{10}\right)^x$$

P (none of the bulbs is defective) $= P(X=0)$

$$= {}^5C_0 \cdot \left(\frac{9}{10}\right)^5 = 1 \cdot \left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$$

Q. 5. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2

(C) 5 (D) $\frac{8}{3}$

Ans. Option (B) is correct.

Explanation :

Let X be the random variable representing a number on the die.

The total number of observations is 6. Therefore,

$$P(X = 1) = \frac{3}{6} = \frac{1}{2}$$

$$P(X = 2) = \frac{2}{6} = \frac{1}{3}$$

$$P(X = 5) = \frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	1/2	1/3	1/6

$$\text{Mean} = E(X) = \sum p_i x_i = \frac{1}{2}\times 1 + \frac{1}{3}\times 2 + \frac{1}{6}\times 5 = \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3+4+5}{6} = \frac{12}{6} = 2$$

Q. 6. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{1}{12}$ (D) $\frac{1}{36}$

Ans. Option (D) is correct.

Explanation : When two dices are rolled, the number of outcomes is 36. The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

$$\therefore \quad E = \{(2, 2)\}$$

$$\Rightarrow P(E) = \frac{1}{36}$$

Q. 7. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

(A) 0.24 (B) 0.3

(C) 0.48 (D) 0.96

Ans. Option (D) is correct.

Explanation :

Here,

$P(A) = 0.4$, $P(B) = 0.8$ and $P(A | B) = 0.6$

$$\because \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \quad P(B \cap A) = P(B|A).P(A) = 0.6 \times 0.4 = 0.24$$

$$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.8 - 0.24 = 1.2 - 0.24 = 0.96$$

Q. 8. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

(A) $\left(\frac{9}{10}\right)^5$ (B) $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(C) $\frac{1}{2}\left(\frac{9}{10}\right)^5$ (D) $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Ans. Option (D) is correct.

Explanation : Here,

$n = 5,\ p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$

$r \le 1$

$\Rightarrow \quad r = 0, 1$

Also,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = P(r = 0) + P(r = 1)$$

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + 5.\frac{1}{10}.\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$$

Q. 9. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is, $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

(A) $\frac{1}{12}$ **(B)** $\frac{1}{40}$

(C) $\frac{13}{120}$ **(D)** $\frac{10}{13}$

Ans. Option (D) is correct.

Explanation : Let E_1 = Event that both A and B solve the problem

$$\therefore \quad P(E_1) = \frac{1}{3}\times\frac{1}{4} = \frac{1}{12}$$

Let E_2 = Event that both A and B got incorrect solution of the problem

$$\therefore \quad P(E_2) = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$$

Let E = Event that they got same answer

Here,

$$P(E/E_1) = 1,$$

$$P(E/E_2) = \frac{1}{20}$$

$$P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)}$$

$$= \frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1) + P(E_2)P(E/E_1)}$$

$$= \frac{\frac{1}{12}\times 1}{\frac{1}{12}\times 1 + \frac{1}{2}\times\frac{1}{20}}$$

$$= \frac{\frac{1}{12}}{\frac{10+3}{120}}$$

$$= \frac{120}{12\times 13}$$

$$= \frac{10}{13}$$

Q. 10. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in physics if she has failed in mathematics is

(A) $\frac{1}{10}$ **(B)** $\frac{2}{5}$

(C) $\frac{9}{20}$ **(D)** $\frac{1}{3}$

Ans. Option (B) is correct.

Explanation : Here,

$$P_{(Ph)} = \frac{30}{100} = \frac{3}{10},$$

$$P_{(M)} = \frac{25}{100} = \frac{1}{4}$$

and $$P_{(M \cap Ph)} = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad P\left(\frac{Ph}{M}\right) = \frac{P(Ph \cap M)}{P(M)} = \frac{\frac{1}{10}}{\frac{1}{4}} = \frac{2}{5}$$

Q. 11. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability, that both cards are queens, is

(A) $\frac{1}{13}\times\frac{1}{13}$ **(B)** $\frac{1}{13}+\frac{1}{13}$

(C) $\frac{1}{13}\times\frac{1}{17}$ **(D)** $\frac{1}{13}\times\frac{4}{51}$

Ans. Option (A) is correct.

Explanation :

Required probability $= \frac{4}{52}\times\frac{4}{52} = \frac{1}{13}\times\frac{1}{13}$

Q. 12. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Ans. Option (C) is correct.

Explanation : Let,

E_1 = Event that the sum of numbers on the dice was less than 6 and

E_2 = Event that the sum of numbers on the dice is 3.

$\therefore\ E_1 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

$\Rightarrow\quad n(E_1) = 10$

and $E_2 = \{(1,2), (2,1)\}$

$\Rightarrow\quad n(E_2) = 2$

$\therefore$ Required probability $= \frac{2}{10}$

$= \frac{1}{5}$

Q. 13. Eight coins are tossed together. The probability of getting exactly 3 heads is

(A) $\frac{1}{256}$ (B) $\frac{7}{32}$

(C) $\frac{5}{32}$ (D) $\frac{3}{32}$

Ans. Option (B) is correct.

Explanation :

We know that, probability distribution

$$P(X = r) = {}^nC_r (p)^r\, q^{n-r}$$

Here, $n = 8,\ r = 3,\ p = \frac{1}{2}$ and $q = \frac{1}{2}$

$$\therefore\ \text{Required probability} = {}^8C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{8-3}$$

$$= \frac{8!}{5!3!}\left(\frac{1}{2}\right)^8$$

$$= \frac{8\cdot 7\cdot 6}{3\cdot 2}\cdot\frac{1}{2^8}$$

$$= \frac{7}{32}$$

Q. 14. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(A) $\frac{3}{28}$ (B) $\frac{2}{21}$

(C) $\frac{1}{28}$ (D) $\frac{167}{168}$

Ans. Option (A) is correct.

Explanation :

Probability of drawing 2 green balls and one blue ball

$$= P(G)\cdot P(G)\cdot P(B) + P(B)\cdot P(G)\cdot P(G) + P(G)\cdot P(B)\cdot P(G)$$

$$= \frac{3}{8}\cdot\frac{2}{7}\cdot\frac{2}{6} + \frac{2}{8}\cdot\frac{3}{7}\cdot\frac{2}{6} + \frac{3}{8}\cdot\frac{2}{7}\cdot\frac{2}{6}$$

$$= \frac{3}{28}$$

Q. 15. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{3}{4}$

Ans. Option (C) is correct.

Explanation : Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore\quad P(E_1) = \frac{3}{6} = \frac{1}{2}$

and $\quad P(E_2) = \frac{13}{52} = \frac{1}{4}$

Then, $\quad P(E_1 \cap E_2) = P(E_1)\cdot P(E_2) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

Q. 16. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4}{7}$

Ans. Option (D) is correct.

Explanation : We have,

$S = \{(B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G, B, B), (B, G, B), (B, B, G)\}$

E_1 = Event that a family has at least one girl, then

$E_1 = \{(G, B, B), (B, G, B), (B, B, G), (G, G, B), (B, G, G), (G, B, G), (G, G, G)\}$

E_2 = Event that the eldest child is a girl, then

$E_2 = \{(G, B, B), (G, G, B), (G, B, G), (G, G, G)\}$

$\therefore \quad E_1 \cap E_2 = \{(G,B,B),(G,G,B),(G,B,G),(G,G,G)\}$

$$\therefore \quad P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{\frac{4}{8}}{\frac{7}{8}} = \frac{4}{7}$$

Q. 17. Two events E and F are independent. If $P(E) = 0.3$, $P(E \cup F) = 0.5$, then $P(E|F) - P(F|E)$ equals

(A) $\frac{2}{7}$ (B) $\frac{3}{35}$

(C) $\frac{1}{70}$ (D) $\frac{1}{7}$

Ans. Option (C) is correct.

Explanation : We have,

$$P(E) = 0.3$$

and $P(E \cup F) = 0.5$

Also, E and F are independent.

Now,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow \quad 0.5 = 0.3 + P(F) - 0.3P(F)$$

$$\Rightarrow \quad P(F) = \frac{0.5 - 0.3}{0.7} = \frac{2}{7}$$

$$\therefore \quad P(E/F) - P(F/E) = P(E) - P(F)$$

(as E and F are independent)

$$= \frac{3}{10} - \frac{2}{7} = \frac{1}{70}$$

Q. 18. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{5}$, then $P(A \cap B)$ equals

(A) $\frac{4}{15}$ (B) $\frac{8}{45}$

(C) $\frac{1}{3}$ (D) $\frac{2}{9}$

Ans. Option (D) is correct.

Explanation : Since A and B are independent events, A' and B' are also independent. Therefore,

$$P(A' \cap B') = P(A') \cdot (B') = (1 - P(A)(1 - P(B)) = \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right) = \frac{2}{5} \cdot \frac{5}{9} = \frac{2}{9}$$

Q. 19. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$, then $P(A|B)$ equals

(A) $1 - P(A|B)$ (B) $1 - P(A|B)$

(C) $\frac{1 - P(A \cup B)}{P(B')}$ (D) $P(A)|P(B)$

Ans. Option (C) is correct.

Explanation : We have,

$$P(A) > 0 \text{ and } P(B) \neq 1$$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{P(B')}$$

Q. 20. Let $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$. Then $P(A|B)$ is equal to

(A) $\frac{6}{13}$ (B) $\frac{4}{13}$

(C) $\frac{4}{9}$ (D) $\frac{5}{9}$

Ans. Option (D) is correct.

Explanation : Here,

$$P(A) = \frac{7}{13},$$

$$P(B) = \frac{9}{13}$$

and $P(A \cap B) = \frac{4}{13}$

$$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{9}{13} \quad \frac{4}{13}}{\frac{9}{13}} = \frac{5}{9}$$

Q. 21. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$. Then $P(B \cap A)$ equals

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) $\frac{3}{10}$ (D) $\frac{1}{5}$

Ans. Option (D) is correct.

Explanation : We have,

$$P(A) = 0.4,$$

$$P(B) = 0.3$$

and $P(A \cup B) = 0.5$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$$

$$P(A) = 0.4,$$
$$P(B) = 0.3$$
and $P(A \cup B) = 0.5$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$

$\therefore$ $P(B' \cap A) = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2 = \frac{1}{5}$

Q. 22. If $P(A) = \frac{2}{5}$ $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'|B').P(B'|A')$ is equal to

(A) $\frac{5}{6}$ (B) $\frac{5}{7}$

(C) $\frac{25}{42}$ (D) 1

Ans. Option (C) is correct.

Explanation : We have,

$$P(A) = \frac{2}{5},$$
$$P(B) = \frac{3}{10}$$
and $P(A \cap B) = \frac{1}{5}$

$$P(A' \mid B') \cdot P(B' \mid A') = \frac{P(A' \cap B')}{P(B')} \cdot \frac{P(A' \cap B')}{P(A')}$$
$$= \frac{(P((A \cup B)'))^2}{P(A')P(B')}$$
$$= \frac{(1 - P(A \cup B))^2}{(1 - P(A))(1 - P(B))}$$
$$= \frac{(1 - P(A) + P(B) - P(A \cap B))^2}{(1 - P(A))(1 - P(B))}$$
$$= \frac{\left[1 - \left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)\right]^2}{\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{10}\right)}$$
$$= \frac{\left(1 - \frac{1}{2}\right)^2}{\frac{3}{5} \cdot \frac{7}{10}}$$
$$= \frac{25}{42}$$

Q. 23. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $(P(B|A)$ is equal to

(A) $\frac{1}{10}$ (B) $\frac{1}{8}$

(C) $\frac{7}{8}$ (D) $\frac{17}{20}$

Ans. Option (C) is correct.

Explanation :

$\because$ $P(A) = \frac{4}{5},$

$P(A \cap B) = \frac{7}{10}$

$\therefore$ $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{7}{8}$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): Let A and B be two events such that $P(A) = \frac{1}{5}$, while $P(A \text{ or } B) = \frac{1}{2}$. Let $P(B) = P$, then for $P = \frac{3}{8}$, A and B independent.

Reason (R) : For independent events,

$$P(A \cap B) = P(A)\,P(B)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A)\,P(B)$$
$$= \frac{1}{5} + P - \left(\frac{1}{5}\right)P$$

$\Rightarrow$ $\frac{1}{2} = \frac{1}{5} + \frac{4}{5}P$

$\Rightarrow$ $P = \frac{3}{8}.$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A) : If A and B are two mutually exclusive events with $P(\bar{A}) = \frac{5}{6}$ and $P(B) = \frac{1}{3}$. Then $P(A / \bar{B})$ is equal to $\frac{1}{4}$.

Reason (R) : If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.2$ then the value of $P(A|\bar{B})$ is 0.2.

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) is correct.
>
> $$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$
>
> $$P(A|\bar{B}) = \frac{P(A)}{P(\bar{B})}$$
>
> [since, given A and B are two mutually exclusive events]
>
> $$P\left(\frac{A}{\bar{B}}\right) = \frac{\left(1-\frac{5}{6}\right).}{\left(1-\frac{1}{3}\right)}$$
>
> $$= \frac{\frac{1}{6}}{\frac{2}{3}}$$
>
> $$= \frac{1}{4}$$
>
> Reason (R) is also correct.
>
> For independent events,
>
> $$P(A|\bar{B}) = P(A) = 0.2.$$

Q. 3. Assertion (A) : Let A and B be two events such that the occurrence of A implies occurrence of B, but not vice-versa, then the correct relation between $P(A)$ and $P(B)$ is $P(B) \geq P(A)$.

Reason (R) : Here, according to the given statement

$$A \subseteq B$$

$$P(B) = P(A \cup (A \cap B)) \quad (\because A \cap B = A)$$

$$= P(A) + P(A \cap B)$$

Therefore, $P(B) \geq P(A)$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 4. Assertion (A) : If $A \subset B$ and $B \subset A$ then, $P(A) = P(B)$.

Reason (R) : If $A \subset B$ then $P(\bar{A}) \leq P(\bar{B})$.

Ans. Option (C) is correct.

> ***Explanation :*** Assertion (A) is correct.
>
> $A \subset B$ and $B \subset A \Rightarrow A = B$
>
> Hence, $P(A) = P(B)$.
>
> But (R) is wrong.
>
> $$A \subset B \Rightarrow \bar{B} \subset \bar{A}$$
>
> Therefore, $P(\bar{A}) \geq P(\bar{B})$

Q. 5. Assertion (A) : The probability of an impossible event is 1.

Reason (R) : If A is a perfect subset of B and $P(A) < P(B)$, then $P(B - A)$ is equal to $P(B) - P(A)$.

Ans. Option (D) is correct.

> ***Explanation :*** Assertion (A) is wrong.
>
> If the probability of an event is 0, then it is called as an impossible event.
>
> But Reason (R) is correct.
>
> From Basic Theorem of Probability,
>
> $P(B - A) = P(B) - P(A)$, this is true only if the condition given in the question is true.

Q. 6. Assertion (A) : If $A = A_1 \cup A_2 \ldots \cup A_n$, where $A_1 \ldots A_n$ are mutually exclusive events then $\sum_{i=1}^{n} P(A_i) = P(A)$

Reason (R) :

Given, $A = A_1 \cup A_2 \ldots \cup A_n$

Since $A_1 \ldots A_n$ are mutually exclusive

$$P(A) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

Therefore $P(A) = \sum_{i=1}^{n} P(A_i)$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. **[CBSE QB 2021]**

Q. 1. Let the target is hit by A, B: the target is hit by B and, C: the target is hit by A and C. Then, the probability that A, B and, C all will hit, is

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

Ans. Option (C) is correct.

Explanation:

$$P(A)=\frac{4}{5},\ P(B)=\frac{3}{4},\ P(C)=\frac{2}{3}$$

Probability that A, B and C all will hit the target

$$= P(A\cap B\cap C)$$
$$= P(A)P(B)P(C)$$
$$= \frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}$$
$$= \frac{2}{5}$$

Q. 2. What is the probability that B, C will hit and A will lose?

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$

(C) $\frac{7}{10}$ (D) $\frac{4}{10}$

Ans. Option (A) is correct.

Explanation:

$$P(\bar{A}) = 1-\frac{4}{5}$$
$$= \frac{1}{5}$$

Probability that B, C will hit and A will lose

$$= P(\bar{A}\cap B\cap C)$$
$$= P(\bar{A}).P(B).P(C)$$
$$= \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}$$
$$= \frac{1}{10}$$

Q. 3. What is the probability that 'any two of A, B and C will hit?

(A) $\frac{1}{30}$ (B) $\frac{11}{30}$

(C) $\frac{17}{30}$ (D) $\frac{13}{30}$

Ans. Option (D) is correct.

Explanation:

$$P(\bar{B}) = 1-\frac{3}{4}$$
$$= \frac{1}{4},$$
$$P(\bar{C}) = 1-\frac{2}{3}$$
$$= \frac{1}{3}$$

Probability that any two of A, B and C will hit

$$= P(\bar{A})P(B)P(C)+P(A)P(\bar{B})P(C)$$
$$+P(A)P(B)P(\bar{C})$$
$$= \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}+\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}$$
$$= \frac{1}{10}+\frac{2}{15}+\frac{1}{5}$$
$$= \frac{3+4+6}{30}$$
$$= \frac{13}{30}$$

Q. 4. What is the probability that 'none of them will hit the target'?

(A) $\frac{1}{30}$ (B) $\frac{1}{60}$

(C) $\frac{1}{15}$ (D) $\frac{2}{15}$

Ans. Option (B) is correct.

Explanation: Probability that none of them will hit the target

$$= P(\bar{A}\cap\bar{B}\cap\bar{C})$$
$$= P(\bar{A}).P(\bar{B}).P(\bar{C})$$
$$= \frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}$$
$$= \frac{1}{60}$$

Q. 5. What is the probability that at least one of A, B or C will hit the target?

(A) $\frac{59}{60}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{1}{60}$

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetecte. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive. **[CBSE QB 2021]**

Q. 1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

(A) 0.001 (B) 0.1

(C) 0.8 (D) 0.9

Ans. Option (D) is correct.

Explanation:

E = Person selected has Covid

F = Does not have Covid

G = Test judge Covid positive

Probability of the person to be tested as Covid positive given that he is actually having Covid

$= P(G/E) = 90\% = \frac{90}{100} = 0.9$

Q. 2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

(A) 0.01 (B) 0.99

(C) 0.1 (D) 0.001

Ans. Option (A) is correct.

Explanation: Probability of person to be tested as Covid positive given that he is actually not having Covid

$= P(G/E) = 1\% = \frac{1}{100} = 0.01$

Q. 3. What is the probability that the 'person is actually not having COVID'?

(A) 0.998 (B) 0.999

(C) 0.001 (D) 0.111

Ans. Option (B) is correct.

Explanation:

$$P(E) = 1 - P(E)$$

$$= 1 - 0.001 \quad \left[\because P(F) = 0.1\% = \frac{0.1}{100} = 0.001\right]$$

$$= 0.999$$

Q. 4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?

(A) 0.83 (B) 0.0803

(C) 0.083 (D) 0.089

Ans. Option (C) is correct.

Explanation:

$$P(E/G) = \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01}$$

$$= \frac{9 \times 10^{-4}}{9 \times 10^{-4} + 99.9 \times 10^{-4}}$$

$$= \frac{9 \times 10^{-4}}{10^{-4}(9 + 99.9)}$$

$$= \frac{9}{108.9}$$

$$= 0.083 \text{ (approx)}$$

Q. 5. What is the probability that the 'person selected will be diagnosed as COVID positive'?

(A) 0.1089 (B) 0.01089

(C) 0.0189 (D) 0.189

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

[CBSE QB 2021]

Q. 1. What is the value of $P(E_1)$?

(A) $\frac{2}{5}$ (B) $\frac{1}{3}$

(C) 1 (D) $\frac{3}{5}$

Ans. Option (D) is correct.

Q. 2. Value of $P(E|E_1)$ is

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{2}{3}$ (D) $\frac{4}{5}$

Ans. Option (B) is correct.

Explanation:

$$P(E_1/E) = 1$$

Q. 3. $\sum_{k=1}^{k=2} P(E|E_k)\, P(E_k)$ Equals

(A) $\frac{11}{15}$ (B) $\frac{4}{15}$

(C) $\frac{1}{5}$ (D) 1

Ans. Option (A) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E|E_k)P(E_K) = P(E|E_1)P(E)+P(E|E_2)+P(E_2)$$

$$= 1\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{5}$$

$$= \frac{11}{15}$$

Q. 4. Value of $\sum_{k=1}^{k=2} P(E_k)$

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$

(C) 1 (D) $\frac{3}{5}$

Ans. Option (C) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E_k) = P(E_1)+P(E_2)$$

$$= \frac{3}{5}+\frac{2}{5}$$

$$= \frac{5}{5}$$

$$= 1$$

Q. 5. What is the probability that the student knows the answer given that he answered it correctly?

(A) $\frac{2}{11}$ (B) $\frac{5}{3}$

(C) $\frac{9}{11}$ (D) $\frac{13}{3}$

Ans. Option (C) is correct.

Explanation:

$$P(E|E_1) = \frac{P(E_1).P(E_1|E)}{P(E_1).P(E_1|E)+P(E_2)P(E|E_2)}$$

$$= \frac{\frac{3}{5}\times 1}{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{3}}$$

$$= \frac{\frac{3}{5}}{\frac{11}{15}}$$

$$= \frac{9}{11}$$

IV. Read the following text and answer the following questions on the basis of the same:

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

[CBSE QB 2021]

Q. 1. The conditional probability that an error is committed in processing given that Sonia processed the form is :

(A) 0.0210 (B) 0.04

(C) 0.47 (D) 0.06

Ans. Option (B) is correct.

Q. 2. The probability that Sonia processed the form and committed an error is :

(A) 0.005 (B) 0.006

(C) 0.008 (D) 0.68

Ans. Option (C) is correct.

Explanation:

P (sonia processed the form and committed an error) = 20% × 0.4

$$= \frac{20}{100}\times 0.04$$

$$= \frac{1}{5}\times 0.04$$

$$= 0.008$$

Q. 3. The total probability of committing an error in processing the form is :

(A) 0 (B) 0.047

(C) 0.234 (D) 1

Ans. Option (B) is correct.

Q. 4. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is :

(A) 1 (B) $\frac{30}{47}$

(C) $\frac{20}{47}$ (D) $\frac{17}{47}$

Ans. Option (D) is correct.

Q. 5. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i|A) = 1$ is :

(A) 0 (B) 0.03

(C) 0.06 (D) 1

Ans. Option (D) is correct.

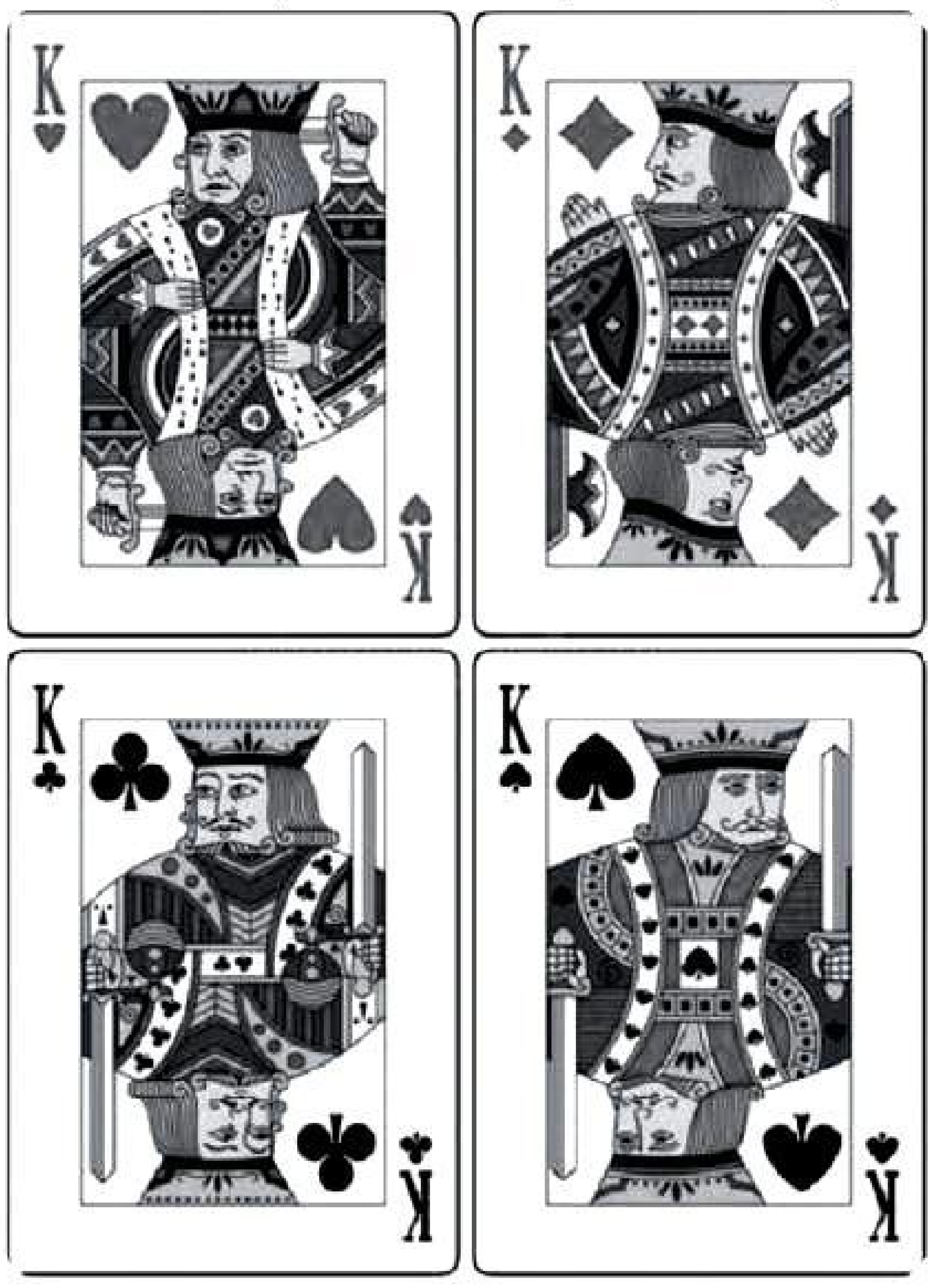
Explanation:

$$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right)+P\left(\frac{E_2}{A}\right)+P\left(\frac{E_3}{A}\right) = 1$$

[∵ sum of all occurrence of an event is equal to 1]

V. Read the following text and answer the following questions on the basis of the same:

A group of people start playing cards. And as we know a well shuffled pack of cards contains a total of 52 cards. Then 2 cards are drawn simultaneously (or successively without replacement).



Q. 1. If x = no. of kings = 0, 1, 2. Then $P(x = 0)$ = ?

(A) $\frac{188}{221}$ **(B)** $\frac{198}{223}$

(C) $\frac{197}{290}$ **(D)** $\frac{187}{221}$

Ans. Option (A) is correct.

Explanation:

$$P(x = 0) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$$

Q. 2. If x = no. of kings = 0, 1, 2. Then $P(x = 1)$ = ?

(A) $\frac{32}{229}$ **(B)** $\frac{32}{227}$

(C) $\frac{32}{221}$ **(D)** $\frac{32}{219}$

Ans. Option (C) is correct.

Explanation:

$$P(x=1) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$$

Q. 3. If x = no. of kings = 0, 1, 2. Then $P(x = 2)$ = ?

(a) $\frac{2}{219}$ **(B)** $\frac{1}{221}$

(C) $\frac{3}{209}$ **(D)** $\frac{1}{209}$

Ans. Option (B) is correct.

Explanation:

$$P(x=2) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

Q. 4. Find the mean of the number of kings ?

(A) $\frac{2}{13}$ **(B)** $\frac{1}{13}$

(C) $\frac{1}{17}$ **(D)** $\frac{2}{17}$

Ans. Option (A) is correct.

Explanation:

$$\text{Mean} = \Sigma x_i p_i$$
$$= \left(0 \times \frac{188}{221}\right) + \left(1 \times \frac{32}{221}\right) + \left(2 \times \frac{1}{221}\right)$$
$$= \frac{34}{221} = \frac{2}{13}$$

Q. 5. Find the variance of the number of kings ?

(A) $\frac{400}{2873}$ **(B)** $\frac{400}{2877}$

(C) $\frac{400}{2879}$ **(D)** $\frac{400}{2871}$

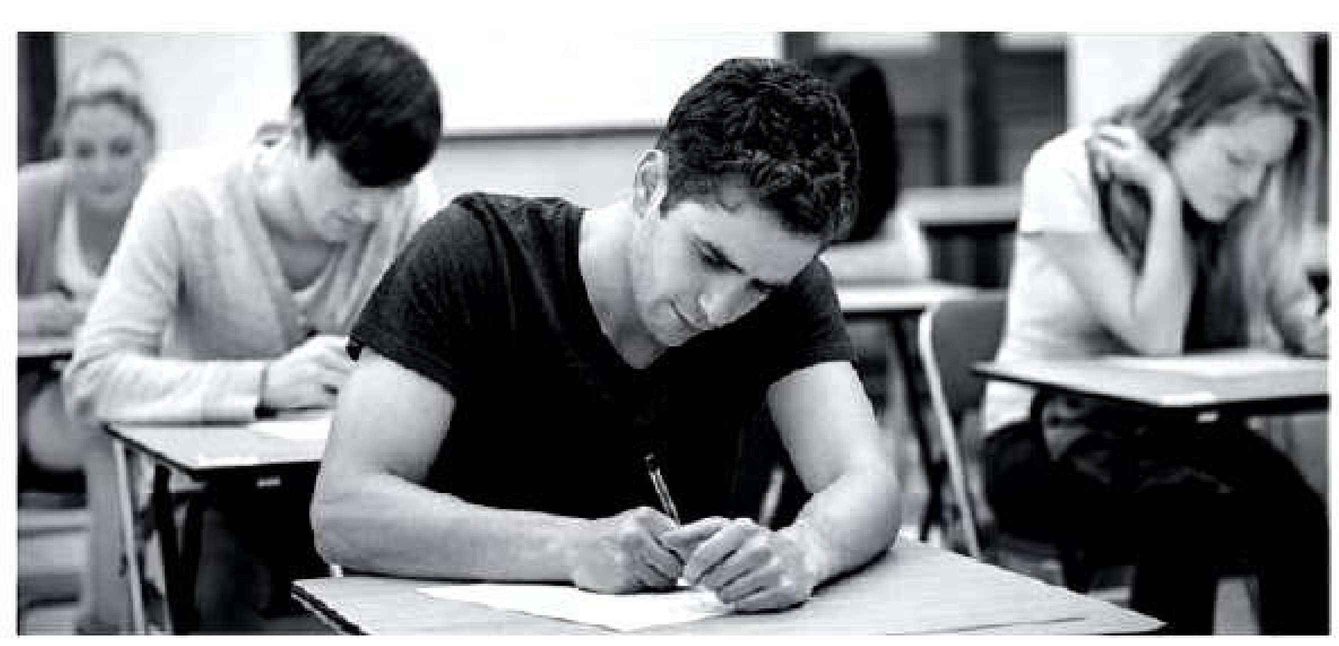
Ans. Option (A) is correct.

Explanation:

$$\text{Variance} = \Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$$
$$\Sigma x_i^2 p_i = \left(0 \times \frac{188}{221}\right) + \left(1 \times \frac{32}{221}\right) + \left(4 \times \frac{1}{221}\right) = \frac{36}{221}$$
$$\text{Variance} = \frac{36}{221} - \left(\frac{2}{13}\right)^2 = \frac{400}{2873}$$

VI. Read the following text and answer the following questions on the basis of the same:

Anand, Samanyu and Shah of SHORTCUTS classes were given a problem in Mathematics whose respective probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. They were asked to solve it independently.



Based on the above data, answer any four of the following questions.

Q. 1. The probability that Anand alone solves it is ________.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{11}{24}$ (D) $\frac{17}{24}$

Ans. Option (A) is correct.

Explanation:

Let $A \rightarrow$ event that Anand solves

$B \rightarrow$ event that Samanyu solves

$C \rightarrow$ event that Shah solves

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}$$

$$\therefore P(A') = \frac{1}{2},\ P(B') = \frac{2}{3},\ P(C') = \frac{3}{4}$$

$$P(A \cap B' \cap C') = P(A)\,P(B')\,P(C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Q. 2. The probability that the problem is not solved is ________.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) 0 (D) $\frac{11}{24}$

Ans. Option (A) is correct.

Explanation:

$$P(A' \cap B' \cap C') = P(A')\,P(B')\,P(C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Q. 3. The probability that the problem is solved is ________.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{17}{24}$ (D) $\frac{11}{24}$

Ans. Option (B) is correct.

Explanation:

$$P(A \cup B \cup C) = 1 - P(A')\,P(B')\,P(C') = 1 - \frac{1}{4} = \frac{3}{4}$$

Q. 4. The probability that exactly one of them solves it is ________.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{17}{24}$ (D) $\frac{11}{24}$

Ans. Option (D) is correct.

Explanation:

$$P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$$

$$= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$$

$$= \frac{11}{24}$$

Q. 5. The probability that exactly two of them solves it is ________.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{17}{24}$ (D) $\frac{11}{24}$

Ans. Option (A) is correct.

Explanation:

$$P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$$

$$= P(A)\,P(B)\,P(C') + P(A)\,P(B')\,P(C) + P(A')\,P(B)\,P(C)$$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$$

$$= \frac{6}{24}$$

$$= \frac{1}{4}$$